A complementary valuation model: Integrating growth dynamics and exit multiples

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Highlights

•We suggest a valuation framework ensuring consistency between entry prices and exit multiples.
•We resolve inherent inconsistencies in traditional DCF valuations by explicitly incorporating growth dynamics and terminal values.
•Our framework enhances valuation reliability for private equity and venture capital investments where exit multiples are crucial.

Abstract

This study suggests a valuation framework resolving inconsistencies in recursive models by integrating growth dynamics and exit multiples. Traditional methods, such as discounted cash flow (DCF) and comparable analysis, often fail to align present valuations with terminal values. Our framework ensures coherence between entry and exit values by integrating intrinsic and relative valuation approaches, providing a consistent tool for complex financial valuations. Particularly suited for venture capital and private equity, our model addresses the logical role of exit multiples in instrinsic valuations.

Keywords: Discounted cash flow; Exit multiples; Growth dynamics; Valuation
JEL codes: G12, G24

1. Introduction

Logical asset valuation is imperative for investors, financial analysts, and corporate strategists. Traditional valuation methodologies, such as intrinsic value models and comparable company analysis, have offered meaningful insights and remain extensively applied in practice. These approaches provide a robust framework for evaluating investment opportunities across diverse industries and market conditions. Nevertheless, evolving market dynamics, shifting growth expectations, and uncertainties surrounding exit values introduce complexities that necessitate supplementary tools. While conventional models retain their relevance, refinements are essential to enhance their practical utility by logically aligning assumptions about future outcomes more closely with present valuations. This is particularly salient in the realms of private equity (Gompers et al. 2016) and venture capital (Gompers et al. 2020), where exit multiples and terminal values significantly influence investment decisions. It is equally relevant for CFOs in corporate finance, who must navigate these challenges when determining appropriate valuations (Graham and Harvey 2001).

A key objective of our valuation model is to preserve the practical insights provided by traditional frameworks, such as discounted cash flow (DCF) and comparable analysis, while addressing specific inconsistencies that may arise when these models are applied to complex financial contexts. Recursive valuation structures often lead to discrepancies between short-term cash flow projections and exit value assumptions, creating a misalignment that can undermine investment decisions. Our framework aims to close that gap by ensuring coherence between entry prices and exit strategies, thereby improving the quality and reliability of valuation outcomes.

In practice, our enhanced model offers a tool for private equity and venture capital professionals, where investment success hinges on accurate forecasting of both interim performance and long-term exit multiples. Similarly, financial analysts operating in corporate finance can benefit from the logical alignment between purchase decisions and the expected cash flow horizon. This alignment ensures that investment strategies remain adaptable across market cycles, providing a stronger foundation for navigating the inherent uncertainties of complex transactions. Ultimately, our model delivers a holistic valuation approach, balancing the short-term metrics essential for day-to-day financial management with the forward-looking considerations critical for exit strategies. This integrated framework equips practitioners with the tools needed to manage sophisticated investments more effectively, bridging the divide between theoretical precision and practical application in dynamic financial markets.

2. Related Literature and Research Background

The traditional DCF model has long served as a foundational tool in asset valuation, widely employed in finance to evaluate investment opportunities, especially under conditions of significant uncertainty (Huang et al. 2023). Based on the principle that an asset's worth is the present value of its expected future cash flows—discounted to reflect time value and associated risks—DCF proves versatile and effective across diverse scenarios. However, the model's sensitivity to key assumptions, particularly regarding growth rates, discount rates, and terminal values, poses challenges in volatile and uncertain markets (Damodaran 2012). Compounding these difficulties, professional sell-side analysts frequently make mistakes when applying DCF, with common errors spanning theoretical misinterpretations, execution flaws, and questionable economic judgments (Green et al. 2016). This raises a pressing question: how can we reduce these mistakes and limitations to establish a more robust and logically sound DCF framework?

To address the limitations of single-stage models, multi-stage DCF frameworks have been introduced, capturing different growth phases and shifts in discount rates over time (Fernandez

2023). These extensions allow for more nuanced projections through differential growth rates across distinct periods. Concurrently, efforts to refine terminal value estimation—by using industry multiples or steady-state assumptions—seek to better represent long-term performance (Harris et al. 2014, Jenkinson et al. 2013, 2020). Despite these improvements, achieving consistency between short-term cash flow projections and terminal value assumptions remains an ongoing challenge.

A key focus in valuation research is the interaction between growth and discount rates; inconsistencies in these parameters can result in significant valuation biases, underscoring the need for coherence across assumptions (Gordon, 1959). Recursive valuation models, which connect present valuations with future expectations, add further complexity, particularly as markets and business environments evolve unpredictably. Further advancements include log-linear approximations, such as the Campbell-Shiller framework, which help to elucidate return predictability (Campbell & Shiller, 1988). While these approaches offer insights into log-linear input-output relationships, their applicability to corporate finance is sometimes limited by real-world complexities.

We propose a structured approach to integrate intrinsic absolute valuation with relative valuation (Chen et al. 2023, Moyer 2005, Sontchi 2012). This is essential, as incorporating relevant industry peer information helps reduce equity valuation errors, with effects differing across firms depending on their information environment and case-specific factors. Furthermore, high-quality industry peer information is associated with enhanced financial performance, primarily due to minimized overvaluation. This information also plays a critical role in reducing unintended wealth transfers between claimants by significantly lowering both the frequency and magnitude of valuation-related errors (Fang et al. 2024).

Though established models remain highly effective and foundational in valuation, they can lack the adaptability needed to align short-term cash flows with long-term exit strategies seamlessly. Our proposed model aims to enhance these frameworks by introducing mechanisms to harmonize present valuations with future outcomes, thereby addressing key inconsistencies identified in the literature.

3. Model

3.1 Motivating example

Existing valuation models often introduce inconsistencies when determining the value of financial assets, particularly in scenarios involving recursive relationships albeit implicitly. A common formulation of such a relationship can be expressed as:

$$P_0 = a_0 + a_1 P_T, \tag{1}$$

$$a_2 = P_T / P_0, \tag{2}$$

where P_t denotes the asset value at t, a_0 captures fundamental components (flow variables such as dividends or cash flows). a_2 represents the exit multiple, encapsulating the influence of future expectations or terminal values. The mathematically correct solution to these equations is:

$$P_0 = a_0/(1-a_1a_2), \tag{3}$$

which ensures that the asset value P_0 remains consistent across both sides of the equation. This formulation guarantees coherence by properly accounting for the recursive nature of asset valuation, ensuring that the final value reflects both fundamental factors and future expectations in an internally consistent manner.

However, traditional valuation methodologies often rely implicitly on the expression $a_0+a_1P_T$ as the starting point for computation. In practice, this is interpreted as:

$$P_0 \leftarrow a_0+a_1P_T, \tag{4}$$

where the asset value on the right-hand side is recursively re-evaluated based on the same expression on the left-hand side. Although this iterative approach can provide a heuristic shortcut, it introduces a subtle but significant inconsistency—there is no guarantee that the computed value of P_0 will converge to the same solution on both sides of the equation. Consequently, this recursive misalignment may distort the valuation process, creating a discrepancy between the fundamental components and future expectations.

The implications of this inconsistency are particularly pronounced in scenarios where terminal values, growth rates, and discount rates interact dynamically. When these variables evolve unpredictably, the recursive nature of the equation can exacerbate misalignments, resulting in an overestimation or underestimation of the true asset value. Such distortions may impair decision-making, particularly in private equity, venture capital, and corporate finance, where consideration for exit multiple plays a critical role. Our model is motivated by the need to address these inconsistencies by proposing a valuation framework that ensures alignment across recursive components. By integrating growth dynamics, terminal values, and discount rates in a coherent manner, the model aims to enhance the accuracy and reliability of asset valuations in complex financial contexts.

In this interpretation, our model can be seen as a generalized extension of the traditional present value framework. In previous models, the assumed terminal value, P_T, directly determines the initial value, P_0. In contrast, our model allows P_T to determine P_0 iteratively, with P_0 in turn informing subsequent values of P_T, and so forth. This iterative process continues until convergence is achieved[1], offering a more dynamic approach that enhances consistency between initial and terminal values.

3.2 Valuation formula suggestion

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[1] Under reasonable conditions, the converged value aligns with our solution.

Consider a hypothetical scenario where a private equity banker evaluates the acquisition of a stake in a firm. Her strategy involves purchasing the stake at the present time, $t = 0$, with the intention of exiting the investment at $t = T$. At the time of exit, the expected price-dividend ratio, PD_T, is anticipated to reflect either the firm's long-run steady-state value or an industry benchmark average (Harris et al. 2014, 2014, Jenkinson et al. 2013). A fundamental question emerges: What constitutes a rational and justifiable purchase price at $t = 0$ that *aligns logically* with the anticipated exit value? To address this question, this paper complements existing valuation frameworks that integrate forward-looking assumptions concerning growth, discount rates, and market conditions, ensuring consistency between the entry price and terminal value.

The asset price is traditionally modeled as the sum of discounted flow variables (e.g., yield, cash flow, coupon payments, or dividends) over T periods. For simplicity, we refer to these flow variables as dividends without loss of generality. Let P_t denote the price of the asset at t. The price can then be expressed as:

$$P_0 = \sum_{i=1}^{T} D_i e^{-ri} + P_T e^{-rT}, \tag{5}$$

where D_i is dividend (flow variable) at time i, r is a potentially time-varying discount rate, P_T is an exit or terminal price at period T.

This formula accounts for the present value of both dividends and the exit value. To facilitate comparison with initial dividends, we divide each side by the current dividend, D_0, and denote g as the (time-varying) growth rate of dividends. The resulting price-dividend ratio, denoted as PD_0, can be expressed as:

$$\mathrm{PD_0} = \sum_{i=0}^{T} e^{(g-r)i} + e^{(g-r)T}\mathrm{PD_T} = \sum_{i=1}^{T} e^{(g-r)i} + \alpha e^{(g-r)T}\mathrm{PD_0}. \tag{6}$$

where g denotes the growth rate of dividends. α is exit multiple, defined as $\alpha = PD_T/PD_0$. The conventional definition of the exit multiple is the ratio of terminal value to initial investment, often expressed as P_T/P_0, such as the multiple of invested capital (MOIC). In this paper, however, we normalize prices by dividends (P/D). Therefore, strictly speaking, α represents the exit multiple in terms of these normalized prices.

The question is whether PD_0 and PD_T are consistent, or equivalently and whether the valuation aligns with exit multiple. α captures the intersection of the intrinsic valuation approach and comparable company analysis. This intersection is crucial, as discounted valuation often transforms into multiple exercises (Mukhlynina and Nyborg 2020).

Alternatively, given that the price-dividend ratio tends to exhibit serial correlation, the parameter α captures the persistence of the ratio over time. Thus, it can be interpreted as:

$$PD_\infty = \alpha PD_t, \tag{7}$$

where the subscript PD_∞ denotes the steady-state ratio, which may serve as a proxy for PD_T in practice, particularly when T is sufficiently large. By rearranging the present value equation, we derive the following expression for the price-dividend ratio:

$$PD = \left(\sum_{i=1}^{T} e^{(g-r)i} \right) \left(1 - \alpha e^{(g-r)T} \right)^{-1} . \tag{8}$$

This formulation ensures consistency between the entry price and the exit value by explicitly accounting for both growth expectations and terminal value assumptions. The model offers a coherent and practical approach for private equity investors, aligning short-term cash flow projections with long-term exit strategies, while addressing the recursive nature of valuation dynamics. If we assume market efficiency, α should be close to one, upon which we can compute fair, under-, or over-valuation of the price-dividend ratio. Figure 1 illustrates the relationship between the price-dividend ratio and the difference between the discount rate and the growth rate, $r–g$. Of course, when the time horizon $T \to \infty$ and assuming $r > g$, the summation converges to:

$$PD = \sum_{i=1}^{\infty} e^{(g-r)i} = \frac{e^{(g-r)}}{1 - e^{(g-r)}} \approx \frac{1}{r-g}. \tag{9}$$

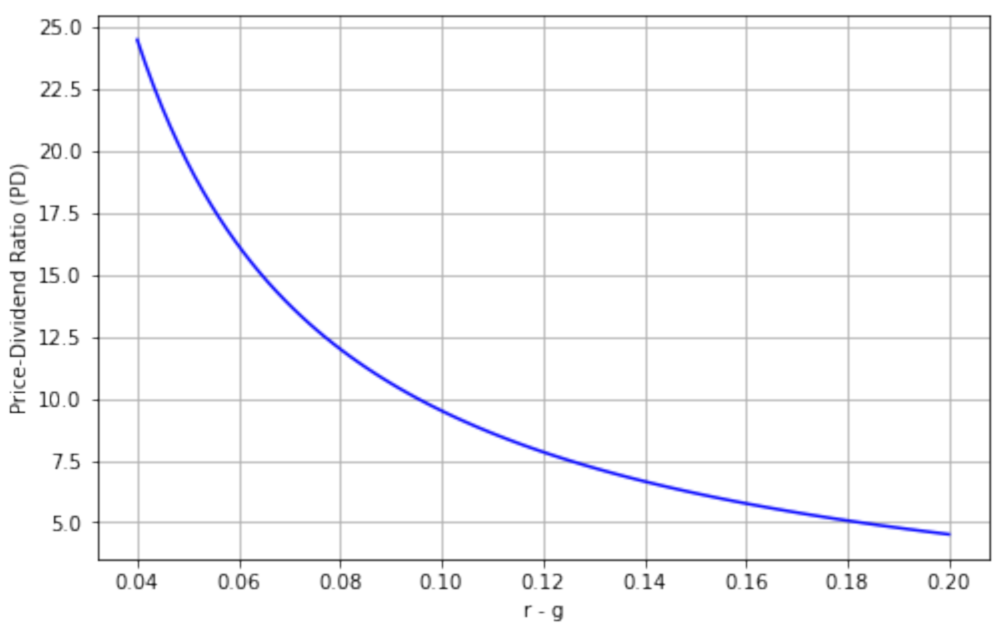

Figure 1. Price-dividend ratio as a function of $r–g$ (α=1, T=5).

Our approach bears some resemblance to the reverse DCF method, a valuation technique that starts with a company's current stock price and works backward to uncover the market-implied assumptions, such as growth rates, profit margins, or discount rates (Cogliati et al. 2011, Paleari

& Vismara 2007, Takács et al. 2020). However, our model aligns more closely with the traditional DCF framework, which involves estimating future cash flows and discounting them to determine a company's intrinsic value. Despite these differences, both approaches emphasize the importance of consistency between exit multiples and underlying assumptions. Our model can be adapted for use in a reverse DCF context, allowing parameters such as the valuation ratio (*PD*) and the multiple (α) to remain fixed while leaving other variables undetermined.

3.3 Lintera approximation

We now undertake a linear approximation by defining $pd \equiv \ln(PD)$. The log transformation allows us to approximate the present value as:

$$pd = \ln\left(\sum_{i=1}^{T} e^{(g-r)i}\right) - \ln\left(1 - \alpha e^{(g-r)T}\right). \tag{10}$$

If we consider a two-period model with T=1, following the Campbell-Shiller framework (Campbell & Shiller 1988), the approximation becomes:

$$pd = \ln\left(e^{g-r}\right) - \ln\left(1 - \alpha e^{g-r}\right). \tag{11}$$

This simplifies to:

$$pd = g - r - \ln(1 - \alpha e^{g-r}). \tag{12}$$

We can interpret $-\ln(1 - \alpha e^{g-r})$ as the modified exit multiple because it increases with α and decreases with *r*-*g*. In addition, the larger *g*, the larger the duration becomes:

$$Duration = -d(pd)/dr = 1 + \alpha e^{g-r}(1 - \alpha e^{g-r})^{-1}. \tag{13}$$

4. Conclusion

We suggest a complementary valuation framework aimed at resolving the inconsistencies present in traditional valuation models, particularly those involving recursive structures and the alignment between entry and terminal values. While established approaches, such as DCF models and comparable multiples, provide valuable insights, they often fall short when applied to scenarios where exit multiples play a crucial role, leading to potential misalignments. Our framework addresses these limitations by explicitly incorporating the recursive nature of valuation into the asset pricing process, ensuring consistency across both sides of the valuation equation. This refinement enhances the reliability of valuations in private equity and venture capital investments, where alignment between purchase prices and terminal exit values is essential for decision-making. The framework's flexibility makes it applicable not only to investment analysis but also to broader strategic financial planning, offering practitioners a coherent tool to manage complex financial scenarios.

We emphasize the importance of aligning key assumptions—such as future growth, discount rates, and exit values—to ensure coherent and robust valuation outcomes. The proposed framework provides a meaningful improvement to conventional models by ensuring logical alignment between entry and terminal values. As financial markets continue to evolve, the ability to navigate uncertainty with refined valuation tools will be critical. This framework offers a reliable basis for managing sophisticated investments, empowering investors to make informed decisions in an increasingly dynamic environment.

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